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FIXED POINTS FOR $\mathscr{A}$–TYPE CONTRACTIVE MAPPING IN G'–METRIC SPACE SATISFYING CONTRACTIVE CONDITION OF INTEGRAL TYPE

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Abstract. In this paper, we obtain fixed point theorems for $\mathscr{A}$-type contractive mappings in G'-metric space satisfying the integral type contractive condition. In last we will give an example to reinforce our main results.

Keywords: metric space; Branciari contractive mapping; $\mathscr{A}$-type contraction; G'-metric space; fixed point result.

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1. Introduction

In nonlinear analysis, fixed point theory plays an important rule for proving the existing theorems of integral and differential equations. In 1922, S. Banach in [2] given an important contractive type mapping for fixed points. For complete metric space this theorem says that, Let (X,δ) be a complete metric space and if $f: X \to X$ is a self mapping and there exists $a \in [0,1)$,

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then f is said to be a contractive mapping such that for all $y,z \in X$,

$$\delta(fy, fz) \leq a \ \ \delta(y,z).$$

Then f has a unique fixed point in X.

Banach contractive condition was generalized by different researchers for different contractive conditions and proved some fixed point theorems that satisfied these generalized contractive conditions For more details we refer [3,5,7,9,10,12–15].

Furthermore, Akram et al. in [1] presented a new type contractive mapping called $\mathscr{A}$-type contraction, and proved some related fixed point results. This class of contractive mapping is super class of Kannan's [5], Bainchini's [3], Reich's [7] type contractions.

In this paper, we obtain fixed point theorems for $\mathscr{A}$-type contractive mappings in $G^{'}$-metric space satisfying the integral type contractive condition. In last we will give an example to support the main results.

Throughout in the existing paper we will use $G^{'}$ for G-metric space and X for complete $G^{'}$-metric space.

2. Preliminaries

In the following section, we give some basic definitions, examples and results on $G^{'}$-metric space.

The concept of $G^{'}$-metric spaces introduced by Mustafa and Sims [6] as follows.

Definition 2.1.[6] Let X be a non empty set and $G^{'} : X \times X \times X \to \mathbb{R}_{+}$, if it satisfy the following properties;

(1) $G^{'}(s,t,u) = 0$ if $s = t = u$,
(2) $G^{'}(s,s,t) > 0$ for all $s,t \in X$,
(3) $G^{'}(s,s,t) \leq G^{'}(s,t,u)$ for all $s,t,u \in X$,
(4) $G^{'}(s,t,u) = G^{'}(s,u,t) = G^{'}(t,u,s) = \dots,$
(5) $G^{'}(s,t,u) \leq G^{'}(s,c,c) + G^{'}(c,t,u)$ for all $s,t,u \in X$.

Then $G^{'}$ is said to be $G^{'}$-metric on X, and $(X, G^{'})$ is called $G^{'}$-metric space.

Note that,

$$d_{G'} = G^{'}(s,t,t) + G^{'}(t,s,s) \quad \text{for all } s,t \in X,$$

this shows that every $G^{'}$-metric is induces a metric $d_{G'}$ on X.

Example 2.2.[6] Consider (X,d) be a metric space and $G^{'} : X \times X \times X \to \mathbb{R}_{+}$ is $G^{'}$-metric on X, defined by

$$G^{'}(s,t,u) = \max\{d(s,t), d(t,u), d(u,s)\},$$

for all $s,t,u \in X$.

Proposition 2.3.[6] Let $(X,G^{'})$ be a $G^{'}$-metric space. Then the following statement are equivalent:

(1) Sequence $\{s_i\}$ is $G^{'}$-convergent to x,

(2) as $i \to \infty$, $G^{'}(s_i,s_i,s) \to 0$,

(3) as $i \to \infty$, $G^{'}(s_i,s,s) \to 0$,

(4) as $i,j \to \infty$, $G^{'}(s_i,s_j,s) \to 0$.

Definition 2.4.[6] Let $(X,G^{'})$ be a $G^{'}$-metric space, and a sequence $\{s_i\}$ is $G^{'}$-Cauchy if for every $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that

$$G^{'}(s_i,s_j,s_k) < \varepsilon \quad \text{for all } i,j,k \in N.$$

that is,

$$G^{'}(s_i,s_j,s_k) \to 0 \quad \text{for all } i,j,k \to \infty.$$

Definition 2.5.[6] A $G^{'}$-metric space is $G^{'}$-complete, if every $G^{'}$-Cauchy is $G^{'}$-convergent in X.

Definition 2.6.[6] Consider a non empty set $\mathscr{A}$ which containing all mappings of α from $\mathbb{R}^3$ to $\mathbb{R}$ satisfying the following conditions.

C_1: α is continuous on the set $\mathbb{R}^3$ (with respect to Euclidean metric on $\mathbb{R}^3$).

C_2: $x \leq ky$ for some $k \in [0,1)$ whenever $x \leq (x,y,y)$ or $x \leq (y,x,y)$ or $x \leq (y,y,x)$ for all $x,y \in \mathbb{R}$.

Definition 2.7.[6] A mapping F on X is said to be $\mathscr{A}$-contraction, if

$$G^{'}(Fs,Ft,Ft) \leq \alpha(G^{'}(s,t,t),G^{'}(s,Fs,Fs),G^{'}(t,Ft,Ft)), \tag{1}$$

for all $s,t \in X$ and for some $\alpha \in \mathscr{A}$.

Branciari in 2002, introduced the general integral type contraction which stated as follows.

Theorem 2.8.[4] *Let (X,d) be a complete metric space, $\delta \in (0,1)$ and let $h : X \to X$ be a mapping such that for each $y,z \in X$,*

$$\int_0^{d(hy,hz)} \psi(z)dz \leq \delta \int_0^{d(y,z)} \psi(z)dz, \tag{2}$$

where ψ from $\mathbb{R}_+$ into $\mathbb{R}_+$ is a Lebesgue-integrable mapping which is summable, nonnegative and such that

$$\int_0^{\varepsilon} \psi(z)dz > 0, \text{ for each } \varepsilon > 0.$$

Then h has a unique fixed point $y \in X$ such that for each $y \in X$, $\lim_{n\to\infty} h^n y = y$.

The result of Branciari was generalized by different researcher to give different contractive condition of integral type. Rhoades in [8] has been done a fine work to give extended form of Branciari integral condition,

$$\int_0^{d(Tx,Ty)} \psi(z)dz \leq a \int_0^{\max\{d(x,y),d(x,Tx),d(y,Ty),\frac{[d(x,Tx)+d(y,,Ty)]}{2}\}} \psi(z)dz$$

for $x,y \in X$ and for some $a \in [0,1)$.

3. Main result

Motivate by the work of Dey et al., [12] we use integral type contractive condition to prove some related fixed points results for $\mathscr{A}$-contraction on $G^{'}$-metric space.

Theorem 3.1. *Let $F : X \to X$ be a self mapping of complete $G^{'}$-metric space satisfying the following condition;*

$$\int_0^{G^{'}(Fs,Ft,Ft)} \phi(z)dz \leq \alpha\Big(\int_0^{G^{'}(s,t,t)} \phi(z)dz, \int_0^{G^{'}(s,Fs,Fs)} \phi(z)dz, \int_0^{G^{'}(t,Ft,Ft)} \phi(z)dz\Big), \tag{3}$$

with some $\alpha \in \mathscr{A}$, where $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is Lebesgue integrable mapping which is summable, nonnegative and such that

$$\int_0^{\varepsilon} \psi(z)dz > 0, \quad \text{for each } \varepsilon > 0. \tag{4}$$

Then F has a unique fixed point $t \in X$, and for each $y \in X$, $\lim_i F^i y = t$.

Proof. Let $t \in X$ be an arbitrary point in $X$and setting $t_{i+1} = Ft_i$, for each $i \geq 1$. Applying the definition of A-type contraction in G-metric space, we get

$$\begin{aligned}
\int_0^{G'(Ft_i,t_{i+1},t_{i+1})} \phi(z)dz &= \int_0^{G'(Ft_{i-1},Ft_i,Ft_i)} \phi(z)dz \\
&\leq \alpha\Big(\int_0^{G'(t_{i-1},t_i,t_i)} \phi(z)dz, \int_0^{G'(t_{i-1},Ft_{i-1},Ft_{i-1})} \phi(z)dz, \\
&\quad \int_0^{G'(t_i,Ft_i,Ft_i)} \phi(z)dz\Big) \\
&\leq \alpha\Big(\int_0^{G'(t_{i-1},t_i,t_i)} \phi(z)dz, \int_0^{G'(t_{i-1},t_i,t_i)} \phi(z)dz, \int_0^{G'(t_i,t_{i+1},t_{i+1})} \phi(z)dz\Big).
\end{aligned} \tag{5}$$

Then by condition (C_2), we get

$$\int_0^{G'(t_i,t_{i+1},t_{i+1})} \phi(z)dz \leq k\int_0^{G'(t_{i-1},t_i,t_i)} \phi(z)dz, \quad \text{for some } k \in [0,1) \text{ as } \alpha \in \mathscr{A}.$$

Continuing in the same way one can get

$$\begin{aligned}
\int_0^{G'(t_i,t_{i+1},t_{i+1})} \phi(z)dz &\leq k\int_0^{G'(t_{i-1},t_i,t_i)} \phi(z)dz \\
&\leq k^2\int_0^{G'(t_{i-2},t_{i-1},t_{i-1})} \phi(z)dz \\
&\quad . \\
&\quad . \\
&\quad . \\
&\leq k^i\int_0^{G'(t_0,t_1,t_1)} \phi(z)dz.
\end{aligned}$$

Letting $i \to \infty$, we get

$$\lim_n \int_0^{G'(t_i,t_{i+1},t_{i+1})} \phi(z)dz \quad = \quad 0,$$

which, from (4) implies that

$$\lim_n G'(t_i,t_{i+1},t_{i+1}) = 0. \tag{6}$$

Now we show that $\{t_i\}$ is a G'-Cauchy sequence in X. Contrary suppose that it is not, and there exists $\varepsilon > 0$ and subsequences i and j such that $i < j < i+1$ with

$$G'(t_i,t_j,t_j) \geq \varepsilon, \quad G'(t_i,t_{j-1},t_{j-1}) < \varepsilon. \tag{7}$$

$$\begin{aligned} G'(t_{i-1},t_{j-1},t_{j-1}) &\leq G'(t_{i-1},t_i,t_i) + G'(t_i,t_{j-1},t_{j-1}) \\ &\leq G'(t_{i-1},t_i,t_i) + \varepsilon. \end{aligned} \tag{8}$$

So by using (**??**) and (8), we get

$$\lim_n \int_0^{G'(t_i,t_{j-1},t_{j-1})} \phi(z)dz \quad \leq \quad \int_0^{\varepsilon} \phi(z)dz. \tag{9}$$

Using (5), (8) and (9) we get

$$\int_0^{\varepsilon} \phi(z)dz \quad \leq \quad \int_0^{G'(t_i,t_j,t_j)} \phi(z)dz \leq \int_0^{G'(t_i,t_{j-1},t_{j-1})} \phi(z)dz \leq k \int_0^{\varepsilon} \phi(z)dz,$$

which is contradiction, as $k \in [0,1)$. Therefore, $\{t_i\}$ is G'-Cauchy sequence and X is G'-complete hence there exists $t \in X$ such that $\{t_i\} \to t$ as $i \to \infty$. From (**??**), we get

$$\begin{aligned} \int_0^{G'(Ft_i,Ft,Ft)} \phi(z)dz &\leq \alpha\Big(\int_0^{G'(t_i,t,t)} \phi(z)dz, \int_0^{G'(t_i,Ft_i,Ft_i)}, \int_0^{G'(t,Ft,Ft)} \phi(z)dz\phi(z)dz \Big) \\ &\leq \alpha\Big(\int_0^{G'(t_i,t,t)} \phi(z)dz, \int_0^{G'(t_i,t_{i+1},t_{i+1})} \phi(z)dz, \int_0^{G'(t,Ft,Ft)} \phi(z)dz \Big). \end{aligned}$$

Taking limit as $i \to \infty$, we get

$$\begin{aligned} \int_0^{G'(t,Ft,Ft)} \phi(z)dz &\leq \alpha\Big(0, \int_0^{G'(t_i,t_{i+1},t_{i+1})} \phi(z)dz, 0\Big) \\ &\leq k.0. \end{aligned}$$

Which implies that $G'(t,Ft,Ft) = 0$, and $t = Ft$.

Next we prove that the fixed point in X is unique, for this suppose $w(\neq t)$ is another fixed point of F. Then from (**??**), we have

$$\begin{aligned}
\int_0^{G'(w,t,t)} \phi(z)dz &= \int_0^{G'(Fw,Ft,Ft)} \phi(z)dz \\
&\leq \alpha\Big(\int_0^{G'(w,t,t)} \phi(z)dz, \int_0^{G'(w,Fw,Fw)} \phi(z)dz, \int_0^{G'(t,Ft,Ft)} \phi(z)dz\Big) \\
&\leq \alpha\Big(\int_0^{G'(w,t,t)} \phi(z)dz, \int_0^{G'(w,w,w)} \phi(z)dz, \int_0^{G'(t,t,t)} \phi(z)dz\Big) \\
&\leq \alpha\Big(\int_0^{G'(w,t,t)} \phi(z)dz, 0, 0\Big) \\
&\leq k.0 \\
&= 0.
\end{aligned}$$

From (4) implies that $G'(w,t,t) = 0$ or, $w = t$. Thus it show that the fixed point is unique. This complete the proof.

4. Common Fixed Points of Two Self Maps and Having Two Related Matrices in Integral Type Contraction

In the following, we illustrate some common fixed point theorems of two self mappings on X and having two related matrices in integral setting.

Theorem 4.1. *Let X be a non empty set with two metrics G' and G'_1 satisfying the following conditions*

1: $\int_0^{G'(s,t,t)} \phi(z)dz \leq \int_0^{G'_1(s,t,t)} \phi(z)dz.$

2: *X is complete with respect to G'.*

3: *F and H are two $\mathscr{A}$-type contractive mappings on X such that F is G–continuous with respect to G' and*

$$\begin{aligned}
\int_0^{G'_1(Fs,Ht,Ht)} \phi(z)dz &\leq \alpha\Big(\int_0^{G'_1(s,t,t)} \phi(z)dz, \int_0^{G'_1(s,Fs,Fs)} \phi(z)dz, \\
&\qquad \int_0^{G'_1(t,Ht,Ht)} \phi(z)dz\Big)
\end{aligned} \tag{10}$$

with some $\alpha \in \mathscr{A}$*, for* $s,t \in X$*, where* $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ *is Lebesgue integrable mapping which is summable (i.e., with finite integral) on each compact subset of* $\mathbb{R}_+$*, non-negative and such that*

$$\int_0^{\varepsilon} \psi(z)dz > 0, \quad \textit{for each } \ \varepsilon > 0. \tag{11}$$

Then F *and* H *have a unique common fixed point* s *in* X*.*

Proof. Consider, a sequence $\{s_n\}$ in X and we define as;

$$s_{2n+1} = Fs_{2n}$$

$$s_{2n} = Hs_{2n-1}.$$

From (10) we have,

$$\begin{aligned}
\int_0^{G_1^{'}(s_1,s_2,s_2)} \phi(z)dz &= \int_0^{G_1^{'}(Fs_0,Hs_1,Hs_1)} \phi(z)dz \\
&\leq \alpha\Big(\int_0^{G_1^{'}(s_0,s_1,s_1)} \phi(z)dz, \int_0^{G_1^{'}(s_0,Fs_0,Fs_0)} \phi(z)dz, \int_0^{G_1^{'}(s_1,Hs_1,Hs_1)} \phi(z)dz\Big) \\
&\leq \alpha\Big(\int_0^{G_1^{'}(s_0,s_1,s_1)} \phi(z)dz, \int_0^{G_1^{'}(s_0,s_1,s_1)} \phi(z)dz, \int_0^{G_1^{'}(s_1,s_2,s_2)} \phi(z)dz\Big) \\
&\leq k\int_0^{G_1^{'}(s_0,s_1,s_1)} \phi(z)dz; \qquad \text{for some } k < 1.
\end{aligned}$$

Similarly, we have

$$\int_0^{G_1^{'}(s_2,s_3,s_3)} \phi(z)dz \;\; \leq \;\; k^2\int_0^{G_1^{'}(s_0,s_1,s_1)} \phi(z)dz.$$

Continuing in this fashion, we can get

$$\int_0^{G_1^{'}(s_i,s_{i+1},s_{i+1})} \phi(z)dz \;\; \leq \;\; k^i\int_0^{G_1^{'}(s_0,s_1,s_1)} \phi(z)dz.$$

Taking limit as $i \to \infty$, we have

$$\lim_i \int_0^{G_1^{'}(s_i,s_{i+1},s_{i+1})} \phi(z)dz = 0,$$

which implies that

$$\lim_i G_1^{'}(s_i,s_{i+1},s_{i+1}) = 0.$$

Then by using condition (C_2), we get

$$\begin{aligned}\int_0^{G'(s_i,s_{i+1},s_{i+1})}\phi(z)dz &\leq \int_0^{G_1'(s_i,s_{i+1},s_{i+1})}\phi(z)dz \\ &\leq k^i\int_0^{G_1'(s_0,s_1,s_1)}\phi(z)dz.\end{aligned}$$

This implies that $G'(s_i,s_{i+1},s_{i+1}) \to 0$, as $i\to\infty$. Now we show that $\{s_i\}$ is a G'–Cauchy sequence in (X,G'), so for any integer $q>0$, we can write

$$\begin{aligned}\int_0^{G'(s_i,s_{i+q},s_{i+q})}\phi(z)dz &\leq \int_0^{G_1'(s_i,s_{i+q},s_{i+q})}\phi(z)dz \\ &\leq \int_0^{G_1'(s_i,s_{i+1},s_{i+1})}\phi(z)dz+\int_0^{G_1'(s_{i+1},s_{i+2},s_{i+2})}\phi(z)dz \\ &\quad +\cdots+\int_0^{G_1'(s_{i+q-1},s_{i+q},s_{i+q})}\phi(z)dz \\ &\leq k^i\int_0^{G_1'(s_0,s_1,s_1)}\phi(z)dz+k^{i+1}\int_0^{G_1'(s_0,s_1,s_1)}\phi(z)dz \\ &\quad +\cdots+k^{i+q-1}\int_0^{G_1'(s_0,s_1,s_1)}\phi(z)dz \\ &\leq \frac{k^i}{1=k}\int_0^{G_1'(s_0,s_1,s_1)}\phi(z)dz. \qquad (12)\end{aligned}$$

Since $k<1$ and as $i\to\infty$, then inequality (12) becomes $\frac{k^i}{1=k}\int_0^{G_1'(s_0,s_1,s_1)}\phi(z)dz\to 0$. Therefore, $\{s_i\}$ is a G'–Cauchy sequence in G'. Hence, $\{s_i\}$ converges to point $s\in X$ by the completeness of G'. Since F is given to be G'–continuous with respect to G' and $s=\lim_{i\to\infty}s_{2i+1}$, we can get $s=\lim_{i\to\infty}Fs_{2i}=F(\lim_{i\to\infty}s_{2i})$. i. e., $s=Fs$. Now,

$$\begin{aligned}\int_0^{G_1'(s,Hs,Hs)}\phi(z)dz &= \int_0^{G_1'(Fs,Hs,Hs)}\phi(z)dz \\ &\leq \alpha\Big(\int_0^{G_1'(s,s,s)}\phi(z)dz,\int_0^{G_1'(s,Fs,Fs)}\phi(z)dz,\int_0^{G_1'(s,Hs,Hs)}\phi(z)dz\Big) \\ &\leq \alpha\Big(0,0,\int_0^{G_1'(s,Hs,Hs)}\phi(z)dz\Big) \\ &\leq k.0 \\ &= 0.\end{aligned}$$

Which gives $s=Hs$. Thus it shows that s is a common fixed point of F and H.

Now, for uniqueness of a common fixed point, let $w(\neq s)$ is another common fixed point of F and H in X. Then, using (10)

$$\begin{aligned}
\int_0^{G_1^{'}(s,w,w)}\phi(z)dz &= \int_0^{G_1^{'}(Fs,Hs,Hs)}\phi(z)dz \\
&\leq \alpha\Big(\int_0^{G_1^{'}(s,w,w)}\phi(z)dz, \int_0^{G_1^{'}(s,Fs,Fs)}\phi(z)dz, \int_0^{G_1^{'}(w,Hw,Hw)}\phi(z)dz\Big) \\
&\leq \alpha\Big(\int_0^{G_1^{'}(s,w,w)}\phi(z)dz, \int_0^{G_1^{'}(s,s,s)}\phi(z)dz, \int_0^{G_1^{'}(w,w,w)}\phi(z)dz\Big) \\
&\leq \alpha\Big(\int_0^{G_1^{'}(s,w,w)}\phi(z)dz, 0, 0\Big) \\
&\leq k.0 \\
&= 0.
\end{aligned}$$

Then by (11) we have $G_1^{'}(s,w,w) = 0$ and $s = w$.

Corollary 4.2. Let X be a non empty set with two metrics G and $G^{'}$ satisfying the following conditions

1: $\int_0^{G(t,s,s)}\phi(z)dz \leq \int_0^{G^{'}(t,s,s)}\phi(z)dz.$

2: X is complete with respect to G.

3: F is $\mathscr{A}$-type contractive mapping on X such that F is $G^{'}$–continuous with respect to $G^{'}$ and

$$\int_0^{G^{'}(Ft,Fs,Fs)}\phi(z)dz \leq \alpha\Big(\int_0^{G^{'}(t,s,s)}\phi(z)dz, \int_0^{G^{'}(t,Ft,Ft)}\phi(z)dz, \int_0^{G(s,Fs,Fs)}\phi(z)dz\Big)$$

with some $\alpha \in \mathscr{A}$, for $t,s \in X$, where $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is Lebesgue integrable mapping which is summable, nonnegative and such that

$$\int_0^{\varepsilon}\psi(z)dz > 0, \quad \text{for each } \varepsilon > 0. \tag{13}$$

Then F has a unique fixed point t in X.

Example 5. Consider, $X = \{0,1,2,3\}$ and $G^{'}$ be the usual $G^{'}$-metric space. Let $F : X \to X$ be a self mapping define by

$$Ft = \begin{cases} 0, & \text{x=2;} \\ 1, & \text{otherwise.} \end{cases}$$

Again let $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ give by $\phi(z) = 1$ for all $z \in \mathbb{R}_+$, Where $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is Lebesgue integrable mapping which is summable, nonnegative and such that

$$\int_0^{\varepsilon} \phi(z)dz > 0, \quad \text{for each } \varepsilon > 0. \tag{14}$$

We know that,

$$\begin{aligned} G^{'}(Fs,Ft,Ft) &\leq \gamma \max\{G^{'}(s,Fs,Fs)+G^{'}(t,Ft,Ft), G^{'}(t,Ft,Ft)+G^{'}(s,t,t),\\ &\quad G^{'}(s,Fs,Fs)+G^{'}(s,t,t)\}, \end{aligned}$$

where $\gamma \in [0, \frac{1}{2})$, is $\mathscr{A}$-contraction. Then we have

$$\begin{aligned} \int_0^{G^{'}(Fs,Ft,Ft)} \phi(z)dz &\leq \alpha\Big(\int_0^{G^{'}(s,t,t)} \phi(z)dz, \int_0^{G^{'}(s,Fs,Fs)} \phi(z)dz, \int_0^{G^{'}(t,Ft,Ft)} \phi(z)dz\Big)\\ &= \gamma \max\Big\{\int_0^{G^{'}(s,Fs,Fs)+G^{'}(t,Ft,Ft)} \phi(z)dz, \int_0^{G^{'}(t,Ft,Ft)+G^{'}(s,t,t)} \phi(z)dz,\\ &\quad \int_0^{G^{'}(s,Fs,Fs)+G^{'}(s,t,t)} \phi(z)dz\Big\}, \end{aligned}$$

for all $s,t \in X$ and for some $\alpha \in \mathscr{A}$ it satisfy the condition of $\mathscr{A}$-contraction. So, all the conditions of Theorem 3.1 is satisfied and hence 1 is the unique fixed point of F. we also can show that our does not satisfy the Branciari and Rhoades contractive conditions Consider $t = 2$ and $s = 1$. Then from Branciari condition for $G^{'}$-metric space we get,

$$\int_0^{G^{'}(Ft,Fs,Fs)} \phi(z)dz \leq a\int_0^{G^{'}(t,s,s)} \phi(z)dz \quad \text{implies } a \geq 1.$$

Which is not satisfy the Branciari integral condition. Again for some $s,t \in X$, we have

$$\begin{aligned} 2 = \int_0^{G^{'}(Ft,Fs,Fs)} \phi(z)dz &\leq a\int_0^{\max\{G^{'}(t,s,s),G^{'}(t,Ft,Ft),G^{'}(s,Fs,Fs),\frac{[G^{'}(t,Ft,Ft)+G^{'}(s,Fs,Fs)]}{2}\}} \phi(z)dz\\ &= \max\{2,0,0,2\}\\ &\leq 2a, \end{aligned}$$

which implies that,

$$a \geq 1.$$

For the value of $a \geq 1$, the Rhoades contractive condition is not satisfy.

Conflict of Interests

The authors declare that there is no conflict of interests.

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