Available online at http://jfpt.scik.org
J. Fixed Point Theory, 2019, 2019:6
ISSN: 2052-5338

CONSTRUCTION OF COMMON RANDOM FIXED POINTS OF A FINITE FAMILY OF $k-$STRICTLY ASYMPTOTICALLY PSEUDOCONTRACTIVE RANDOM OPERATORS WITH ERRORS IN ARBITRARY REAL BANACH SPACES

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Abstract. We prove that the recent result of Banaejee and Choudhury [Commun. Korean Mathematical Society, 1(26)(2011), 23-35] concerning the random iterative approximation of common random fixed points of a finite family of asymptotically nonexpansive random operators in arbitrary real Banach spaces using composite implicit random iteration method with errors can be extended to the much more general class of $k-$strictly asymptotically pseudocontractive random operators. Our result complements and generalizes many other results in literature.

Keywords: common random fixed points; $k-$strictly asymptotically pseudocontractive random operator; Iterative approximation; arbitrary Banach spaces.

2010 AMS Subject Classification: 47H10, 47J05, 65J15.

1. Introduction

Let K be a nonempty subset of an arbitrary real Banach space E and J the normalized duality mapping from E into 2^{E^*} given by

$$j(x) = \{f \in E^* : \langle x, f\rangle = \|x\|^2 : \|x\|^2 = \|f\|^2\}$$

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Received February 3, 2019

where E^* denotes the dual space of E and $\langle .,. \rangle$ denotes the generalized duality pairing. If E^* is strictly convex, then J is single-valued. In the sequel, we shall denote single-valued duality mapping by j.

The development of random fixed point iterations was initiated by Choudhury in [] where random Ishikawa iteration scheme was defined and its strong convergence to a random fixed point in Hilbert spaces was discussed. After that several authors have worked on random fixed point iterations some of which are noted in [5, 6, 9, 12, 13, 14, 28, 29].

Banaejee and Choudhury [5] constructed and studied different random iterative algorithms for weakly contractive and asymptotically nonexpansive random operators on arbitrary Banach spaces. They also established the convergence of an implicit random iteration process to a common random fixed point for a finite family of asymptotically quasi-nonexpansive random operators. Very recently Plubtieng *et al.* [28] constructed and established the convergence of an implicit random iteration process with errors for a common random fixed point of a finite family of asymptotically quasi-nonexpansive random operators in the setting of uniformly convex Banach spaces

Recently, Banaejee and Choudhury [5], introduced the following Random iteration scheme and called it Composite Implicit Random Iteration Method. From $\xi_1(t) \in K$, the sequence $\{\xi_n(t)\}_{n=1}^{\infty}$ is generated by

$$\left.\begin{aligned} \xi_n(t) &= \alpha_n \xi_{n-1}(t) + \beta_n T_{i(n)}^{k(n)}(t, \eta_n(t)) + \gamma_n f_n(t) \\ \eta_n(t) &= a_n \xi_n + b_n T_{i(n)}^{k(n)}(t, \xi_n(t)) + c_n g_n(t) \end{aligned}\right\}, \quad n \geq 1, \quad t \in \Omega \tag{1}$$

where $1 \leq n = (k(n)-1)N + i(n), i = \{1,2,...,N\}, \{\alpha_n\}_{n=1}^{\infty}, \{\beta_n\}_{n=1}^{\infty}, \{\gamma_n\}_{n=1}^{\infty}$, $\{a_n\}_{n=1}^{\infty}, \{b_n\}_{n=1}^{\infty}, \{c_n\}_{n=1}^{\infty} \subseteq [0,1], \quad T_n = T_{(n-N)modN}$ and they proved the convergence of a composite implicit random iterative scheme with errors for a finite family of asymptotically nonexpansive random operators in Banach spaces. Observe that if $T : K \longrightarrow K$ is uniformly $L-$Lipschitzian $k-$strictly asymptotically pseudocontractive random operators with a sequence $\{a_n\}_{n=1}^{\infty}$ such that $\lim_{n\to\infty} a_n = 1$, then for every fixed $u \in K$ and $t \in (\frac{1}{(1+L)}, 1)$, the operator $S_{t,s,n} : K \longrightarrow K$ defined for all $x \in K$ by $S_{t,s,n}x = tu + (1-t)T^n[su + (1-s)T^n x]$ satisfies $\|S_{t,s,n}x - S_{t,s,n}y\| \leq (1-t)(1-s)L^2\|x-y\| \forall x, y \in K$. Thus, the composite implicit random iteration process (1) is defined in K for the family $\{T_i\}_{i=1}^{N}$ of N uniformly $L-$ Lipschitzian $k-$strictly asymptotically pseudocontractive random operators of the nonempty closed convex subset K of a real Banach space provided that $\{\alpha_n\}_{n=1}^{\infty}, \{\beta_n\}_{n=1}^{\infty}, \{\gamma_n\}_{n=1}^{\infty}, \{a_n\}_{n=1}^{\infty}, \{b_n\}_{n=1}^{\infty}, \{c_n\}_{n=1}^{\infty} \subseteq [0,1] \forall n \geq 1$ and $L = \max_{1\leq i\leq N}\{L_i\}$.

In this paper, we prove that the iteration process (1) converges strongly to the common fixed points of the finite family of N uniformly $L-$Lipschitzian $k-$strictly asymptotically pseudocontractive random operators in arbitrary real Banach spaces. Our results generalize theorem 1.1 and extend the recent result of Banaejee and Choudhury [5] from asymptptically nonexpansive random operators to the much more general $k-$strictly asymptotically pseudocontractive random operators. In the sequel, we need the following.

Definition 1.1: Let (Ω, Σ) denotes a measurable space and E stands for real Banach space. For any function $T : \Omega \times E \longrightarrow E$ we denote the $n-th$ iterate $T(t, T(t, ..., T(t,x)))$ of T by $T^n(t,x)$. A function $T : \Omega \longrightarrow E$ is said to be measurable if $f^{-1}(B) \in \Sigma$ for every Borel subset B of E.

Definition 1.2: An operator $T : \Omega \times E \longrightarrow E$ is called random operator if $T(.,x) : \Omega \longrightarrow E$ is measurable for every $x \in E$.

Definition 1.3: A random operator $T : \Omega \times E \longrightarrow E$ continuous if $T(t,.) : \Omega \longrightarrow E$ is continuous for each $t \in \Omega$.

Definition 1.4: A measurable function $p : \Omega \longrightarrow E$ is said to be random fixed point of random operator $T : \Omega \times E \longrightarrow E$ if $T(t, p(t)) = p(t)$, $t \in \Omega$. The set of all random fixed points of T is denoted by $RF(T)$. Definition 1.5: Let K be nonempty subset of a separable Banach space E and $T : \Omega \times K \longrightarrow K$ be random operator. Then T is said to be

(i) Nonexpansive random operator

$$\|T(t,x) - T(t,y)\| \leq \|x - y\|, \tag{2}$$

$\forall x, y \in K$ and for each $t \in \Omega$.

(ii) Asymptotically Nonexpansive random operator if there exists a sequence of measurable function $r_n(t) : \Omega \longrightarrow [1, \infty)$ with $\lim_{n\to\infty} r_n(t) = 1$ for each $t \in \Omega$ such that

$$\|T^n(t,x) - T^n(t,y)\| \leq r_n(t)\|x - y\|, \tag{3}$$

$\forall x, y \in K, n \in N$ and for each $t \in \Omega$.

(iii) Asymptotically quasi-nonexpansive random operator if there exists a sequence of measurable function $r_n(t) : \Omega \longrightarrow [0,\infty)$ with $\lim_{n\to\infty} r_n(t) = 0$ for each $t \in \Omega$ such that

$$\|T^n(t,\eta_n(t)) - p(t)\| \leq (r_n(t)+1)\|\eta_n(t) - p(t)\|, \tag{4}$$

$\forall x, y \in K, n \in N$ and for each $t \in \Omega$. where $p : \Omega \longrightarrow K$ is a random fixed point of T and $\eta : \Omega \longrightarrow K$ any measurable map.

(iv) Uniformly $L-$Lipschitzian random operator if for any $x, y \in K$ and for each $t \in \Omega$ such that

$$\|T^n(t,x) - T^n(t,y)\| \leq L\|x - y\|, \tag{5}$$

where $n \geq 1$ and $L > 0$.

(v) semicompact random operator if for sequence of measurable mappings $\{\xi_n\}$ from Ω to K, with $\lim_{n\to\infty} \|\xi_n(t) - T(t,\xi_n(t))\| = 0$ $\forall t \in \Omega$, we have a subsequence $\{\xi_{n_k}\} \subseteq \{\xi_n\}$ such that $\xi_{n_k} \to \xi^* \in K \forall t \in \Omega$, where ξ is a measurable mappings $\{\xi_n\}$ from Ω to K.

Definition 1.6 $k-$strictly asymptotically pseudocontractive random operators if there exists a sequence of measurable function $r_n(t) : \Omega \longrightarrow [1,\infty)$ with $\lim_{n\to\infty} r_n(t) = 1$ for each $t \in \Omega$ if $\forall \xi, \eta \in K, \exists j(\xi_n(t) - \eta(t)) \in J(\xi_n(t) - \eta(t))$ and a constant $k \in (0,1)$ such that such that

$$\begin{aligned} \langle (I - T_{i(n)}^{k(n)})(t,\xi_n(t)) - (I - T_{i(n)}^{k(n)})(t,\eta_n(t)), j(\xi_n(t) - \eta(t))\rangle \geq \\ \frac{1}{2}(1-k)\|\xi_n(t) - T_{i(n)}^{k(n)}(t,\xi_n(t)) - \eta_n(t) - T_{i(n)}^{k(n)}(t,\eta_n(t)\|^2 \\ -\frac{1}{2}(r_{in} - 1)\|\xi_n(t) - \eta_n(t)\|^2, \forall n \in N. \end{aligned} \tag{6}$$

Definition 1.7 [31]: A bounded convex subset K of a real Banach space E is said to have normal structure if every nontrivial convex subset C of K contains at least one nondimentrial point. That is, $\exists \quad x_0 \in E$ such that $\sup\{\|x_0 - x\| : x \in C\} < \sup\{\|x - y\| : x, y \in C = d(C)\}$ where $d(C)$ is the diameter of C.

Every uniformly convex Banach space and every compact convex subset K of a Banach space E has normal structure. For the definition of modulus of convexity of E and the characteristic of convexity ε_0 of E, see (15).

Definition 1.8 A finite family $\{T_i : i \in I\}$ of N continuous random operators from $\Omega \times K \longrightarrow$

K with $F = \bigcap_{n=1}^{N} RF(T_i) \neq \emptyset$ is said to satisfy condition (A) if $F(T) \neq \emptyset$ and there exist a nondecreasing functions $f : [0,\infty) \to [0,\infty)$ and $g : [0,\infty) \to [0,\infty)$ with $f(0) = 0, \quad f(r) > 0 \quad \forall r \in (0,\infty)$ and $\forall t \in \Omega$ such that $f(d(x(t),F(T))) \leq \max_{1\leq i\leq N}\{\|x(t) - T_i(t,x(t))\|\}$ where $x \in K$. Lemma 1.1 ([25] , p. 80): Let $\{a_n\}_{n=1}^{\infty}, \{b_n\}_{n=1}^{\infty}$ *and* $\{\delta_n\}_{n=1}^{\infty}$ be sequences of nonnegative real numbers satisfying the inequality $a_{n+1} \leq (1+\delta_n)a_n + b_n, n \geq 1$. If $\sum_{n=1}^{\infty} \delta_n < \infty$ and $\sum_{n=1}^{\infty} b_n < \infty$, then $\lim_{n\to\infty} a_n$ exists. If in addition $\{a_n\}_{n=1}^{\infty}$ has a subsequence which converges strongly to zero, then $\lim_{n\to\infty} a_n = 0$. Lemma 1.2 ([31], Page 829-830). Let E be an arbitrary normed space and let $\{t_n\}_{n=1}^{\infty}$ be a real sequence satisfying the conditions: Let E be an arbitrary normed space and let $\{t_n\}_{n=1}^{\infty}$ be a real sequence satisfying the conditions;

(i) $0 \leq t_n \leq t \leq 1$, and for some $t \in (0,1)$,

(ii) $\Sigma_{n=1}^{+\infty} t_n = +\infty$,

Let $\{u_n\}_{n=1}^{\infty}$ and $\{v_n\}_{n=1}^{\infty}$ be two sequences in E such that

(iii) $u_{n+1} = (1-t_n)u_n + t_n v_n, \quad \forall n \geq 1$

(iv) $\lim_{n\to\infty} \|u_n\| = d$ for some $d \in [0,\infty)$,

(v) $\limsup_{n\to\infty} \|v_n\| \leq d$,

(vi) $\{\Sigma_{j=1}^{n} t_j v_j\}_{n=1}^{+\infty}$ is bounded. Then $d = 0$.

Theorem 1.2 ([1], Corollary 3.6). Let E be a real Banach space with normal structure $N(E) > \max(1,\varepsilon_0), \varepsilon_0 > 0, K$ a nonempty bounded closed convex subset of E and $T : K \to K$ a uniformly $L-$Lipschitzian mapping with $L < \alpha, \alpha > 1$. Then T has a fixed point. We now prove the following main results:

2. Main Results

Theorem 2.1: Let E be a real Banach spaces and K a nonempty closed convex subset of E. Let $\{T_i : i \in \mathscr{I}\}$ be N uniformly $k-$strictly asymptotically pseudocontractive random operators $\Omega \times K$ to K with the sequence of measurable mappings $\{r_{in}\} : \Omega \to [1,\infty)$ satisfying $\sum_{n=1}^{\infty}(r_{in}(t) - 1) < \infty$ for $t \in \Omega$ and $\forall i \in \mathscr{I} = \{1,2,...,N\}$. Suppose that $F = \bigcap_{n=1}^{N} RF(T_i) \neq \emptyset$. Let $\{\xi_n\}$ be the implicit random iterative sequence with errors defined by (1) with additional assumptions: (i) $0 < \alpha < \alpha_n, \quad \beta_n \leq \beta < 1$ (ii) $\sum_{n=1}^{\infty} \beta_n = +\infty$ (iii) $\sum_{n=1}^{\infty} \beta_n^2 < +\infty$ (iv) $\sum_{n=1}^{\infty} b_n^2 < +\infty$ (v) $\sum_{n=1}^{\infty} c_n^2 < +\infty$ (vi) $\sum_{n=1}^{\infty} \gamma_n^2 < +\infty$. Then

(a) $\lim_{n\to\infty}\|\xi_n(t)-p(t)\|$ exists for each $p\in F(T)$ and $t\in\Omega$,

(b) $\lim_{n\to\infty}\|\xi_n(t)-T_i(t,\xi_n(t)\|=0$,

(c) $\{\xi_n(t)\}_{n=1}^{\infty}$ converges strongly to a common random fixed points of the random operators $\{T_i : i\in\mathscr{I}\}$ if and only if there exists a subsequence $\{\xi_{n_j}(t)\}_{j=1}^{\infty}$ of $\{\xi_n(t)\}_{n=1}^{\infty}$ which converges strongly to $p(t)$ of $\{T_i\}_{n=1}^{N}$.

Proof: It is well known (see for example Chang [8]) that the inequality

$$\|x+y\|^2 \le \|x\|^2 + 2\langle y, j(x+y)\rangle \tag{7}$$

holds for $x,y\in E$ and $j(x-y)\in J(x-y)$. Let $p(t)\in F$, then using (1) and (7), we have,

$$\begin{aligned}
&\|\xi_n(t)-p(t)\|^2 = \|(\alpha_n\xi_{n-1}(t)+\beta_nT_{i(n)}^{k(n)}(t,\eta_n(t))+\gamma_nf_n(t))-p(t)\|^2\\
=\;& \|\alpha_n(\xi_{n-1}(t)-p(t))+\beta_n(T_{i(n)}^{k(n)}(t,\eta_n(t))-p(t))+\gamma_n(f_n(t)-p(t))\|^2\\
\le\;& \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+2\beta_n\langle T_{i(n)}^{k(n)}(t,\eta_n(t))-p(t),j(\xi_n(t)-p(t))\rangle\\
&+2\gamma_n\langle f_n(t)-p(t),j(\xi_n(t)-p(t)\rangle\\
=\;& \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+2\beta_n\langle T_{i(n)}^{k(n)}(t,\eta_n(t))\\
&-T_{i(n)}^{k(n)}(t,\xi_n(t)),j(\xi_n(t)-p(t))\rangle\\
&+2\beta_n\langle T_{i(n)}^{k(n)}(t,\xi_n(t))-p(t),j(\xi_n(t)-p(t))\rangle+2\gamma_n\langle f_n(t)\\
&-p(t),j(\xi_n(t)-p(t)\rangle\\
\le\;& \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+2\beta_n\|T_{i(n)}^{k(n)}(t,\eta_n(t))-T_{i(n)}^{k(n)}(t,\xi_n(t))\|\|\xi_n(t)-p(t)\|\\
&+2\beta_n\langle T_{i(n)}^{k(n)}(t,\xi_n(t))-p(t),j(\xi_n(t)-p(t))\rangle+2\gamma_n\|f_n(t)-p(t)\|\\
&\times\|\xi_n(t)-p(t)\|\\
\le\;& \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+2\beta_nL\|\eta_n(t)-\xi_n(t)\|\|\xi_n(t)-p(t)\|\\
&+2\beta_n\langle T_{i(n)}^{k(n)}(t,\xi_n(t))-\xi_n(t),j(\xi_n(t)-p(t))\rangle\\
&+2\beta_n\langle\xi_n(t)-p(t),j(\xi_n(t)-p(t))\rangle+2\gamma_n\|f_n(t)-p(t)\|\|\xi_n(t)-p(t)\|\\
=\;& \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+2\beta_nL\|\eta_n(t)-\xi_n(t)\|\|\xi_n(t)-p(t)\|\\
&-2\beta_n\langle\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t)),j(\xi_n(t)-p(t))\rangle\\
&+2\beta_n\|\xi_n(t)-p(t)\|^2+2\gamma_n\|f_n(t)-p(t)\|\|\xi_n(t)-p(t)\| \qquad (8)
\end{aligned}$$

$$
\begin{aligned}
\|\eta_n(t) \quad - \quad & \xi_n(t)\| = \|(a_n\xi_n(t)+b_nT_{i(n)}^{k(n)}(t,\xi_n(t))+c_ng_n(t))-\xi_n(t)\| \\
= \quad & \|a_n(\xi_n(t)-\xi_n(t))+b_n(T_{i(n)}^{k(n)}(t,\xi_n(t)-\xi_n(t))+c_n(g_n(t)-\xi_n(t))\| \\
\leq \quad & b_n\|T_{i(n)}^{k(n)}(t,\xi_n(t)-\xi_n(t)\|+c_n\|f_n(t)-\xi_n(t)\| \\
= \quad & b_n\|T_{i(n)}^{k(n)}(t,\xi_n(t)-p(t)+p(t)-\xi_n(t)\| \\
& +c_n\|g_n(t)-p(t)+p(t)-\xi_n(t)\| \\
\leq \quad & b_nL\|\xi_n(t)-p(t)\|+b_n\|\xi_n(t)-p(t)\|+c_n\|g_n(t)-p(t)\| \\
& +\|\xi_n(t)-p(t)\| \\
= \quad & (b_nL+b_n+c_n)\|\xi_n(t)-p(t)\|+c_n\|g_n(t)-p(t)\| \qquad (9)
\end{aligned}
$$

Substitute (9) into (8)

$$
\begin{aligned}
\|\xi_n(t) \quad - \quad & p(t)\|^2 \leq \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+2\beta_nL\{(b_nL+b_n+c_n)\|\xi_n(t)-p(t)\| \\
& +c_n\|g_n(t)-p(t)\|\}\|\xi_n(t)-p(t)\| \\
& -2\beta_n\langle\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t)),j(\xi_n(t)-p(t))\rangle \\
& +2\beta_n\|\xi_n(t)-p(t)\|^2+2\gamma_n\|f_n(t)-p(t)\|\|\xi_n(t)-p(t)\| \\
= \quad & \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+2\beta_nL(b_nL+b_n+c_n)\|\xi_n(t)-p(t)\|^2 \\
& +2\beta_nc_nL\|g_n(t)-p(t)\|\|\xi_n(t)-p(t)\| \\
& -2\beta_n\langle\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t)),j(\xi_n(t)-p(t))\rangle \\
& +2\beta_n\|\xi_n(t)-p(t)\|^2+2\gamma_n\|f_n(t)-p(t)\|\|\xi_n(t)-p(t)\|
\end{aligned}
$$

Using the fact that $2\|x\|\|y\| \leq \|x\|^2+\|y\|^2$, then

$$
\begin{aligned}
\|\xi_n(t) \quad - \quad & p(t)\|^2 \leq \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+2\beta_nL(b_nL+b_n+c_n)\|\xi_n(t)-p(t)\|^2 \\
& +2\beta_nc_nL\|g_n(t)-p(t)\|\|\xi_n(t)-p(t)\| \\
& -2\beta_n\langle\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t)),j(\xi_n(t)-p(t))\rangle \\
& +2\beta_n\|\xi_n(t)-p(t)\|^2+\gamma_n\|f_n(t)-p(t)\|^2+\gamma_n\|\xi_n(t)-p(t)\| \\
= \quad & \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+[2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n] \\
& \times\|\xi_n(t)-p(t)\|^2+\beta_nc_nL\|g_n(t)-p(t)\|^2 \\
& -2\beta_n\langle\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t)),j(\xi_n(t)-p(t))\rangle \\
& +\gamma_n\|f_n(t)-p(t)\|^2 \qquad (10)
\end{aligned}
$$

Since $T_{i(n)}: K \to K$ is $k-$strictly asymptotically pseudocontractive random operators, we have

$$\begin{aligned}
\|\xi_n(t) - p(t)\|^2 &\leq \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2 + [2\beta_n L(b_nL+b_n+c_n)+\beta_n c_n L \\
&\quad +\gamma_n+2\beta_n]\|\xi_n(t)-p(t)\|^2+\beta_n c_n L\|g_n(t)-p(t)\|^2 \\
&\quad -\beta_n(1-k)\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2+\beta_n(r_{in}-1)\|\xi_n(t)-p(t)\|^2 \\
&\quad +\gamma_n\|f_n(t)-p(t)\|^2 \\
&= \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+[2\beta_n L(b_nL+b_n+c_n)+\beta_n c_n L \\
&\quad +\gamma_n+2\beta_n+\beta_n(r_{in}-1)]\|\xi_n(t)-p(t)\|^2 \\
&\quad +\beta_n c_n L\|g_n(t)-p(t)\|^2-\beta_n(1-k)\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2 \\
&\quad +\gamma_n\|f_n(t)-p(t)\|^2 \qquad (11)
\end{aligned}$$

Since $\|f_n(t)-p(t)\| \leq M(t)$ and $\|g_n(t)-p(t)\| \leq M(t)$, then

$$\begin{aligned}
\|\xi_n(t) - p(t)\|^2 &\leq \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2 + [2\beta_n L(b_nL+b_n+c_n)+\beta_n c_n L \\
&\quad +\gamma_n+2\beta_n+\beta_n(r_{in}-1)]\|\xi_n(t)-p(t)\|^2+\beta_n c_n L M(t) \\
&\quad -\beta_n(1-k)\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2+\gamma_n M(t)
\end{aligned}$$

$$\begin{aligned}
&1-[2\beta_n L(b_nL+b_n+c_n)+\beta_n c_n L+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]\|\xi_n(t)-p(t)\|^2 \\
\leq\ & \alpha_n^2\|\xi_{n-1}(t)-p(t)\|^2+\beta_n c_n L M(t) \\
&-\beta_n(1-k)\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2+\gamma_n M(t) \qquad (12)
\end{aligned}$$

$$\begin{aligned}
\|\xi_n(t) - p(t)\|^2 &\leq \frac{\alpha_n^2}{1-[2\beta_n L(b_nL+b_n+c_n)+\beta_n c_n L+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}\|\xi_{n-1}(t)-p(t)\|^2 \\
&\quad -\frac{\beta_n(1-k)}{1-[2\beta_n L(b_nL+b_n+c_n)+\beta_n c_n L+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2 \\
&\quad +\frac{\beta_n c_n L+\gamma_n}{1-[2\beta_n L(b_nL+b_n+c_n)+\beta_n c_n L+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}M(t)
\end{aligned}$$

$$\|\xi_n(t) - p(t)\|^2 \leq \left\{1+\frac{\alpha_n^2-1+2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n+\beta_n(r_{in}-1)}{1-[2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}\right\} \times\|\xi_{n-1}(t)-p(t)\|^2 - \frac{\beta_n(1-k)}{1-[2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2 + \frac{\beta_nc_nL+\gamma_n}{1-[2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}M(t)$$

$$\|\xi_n(t) - p(t)\|^2 \leq \left\{1+\frac{\beta_n^2+2\beta_n\gamma_n-\gamma_n(1-\gamma_n)+2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\beta_n(r_{in}-1)}{1-[2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}\right\} \times\|\xi_{n-1}(t)-p(t)\|^2 - \frac{\beta_n(1-k)}{1-[2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2 + \frac{\beta_nc_nL+\gamma_n}{1-[2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n+\beta_n(r_{in}-1)]}M(t) \tag{13}$$

Since $1-[2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL+\gamma_n+2\beta_n+\beta_n(r_{in}-1)] = 1-\beta_n[2L(b_nL+b_n+c_n)+c_nL+2+(r_{in}-1)]-\gamma_n$, from condition (i), $\lim_{n\to\infty}\beta_n=0$. So there exists a natural number N_1 such that $\forall\ n \ \geq\ N_2$,

$$1-2\beta_nL(b_nL+b_n+c_n)-\beta_nc_nL-\gamma_n-2\beta_n-\beta_n(r_{in}-1) \geq \frac{1}{2} \tag{14}$$

So that

$$\|\xi_n(t) - p(t)\|^2 \leq [1+2\{\beta_n^2+2\beta_n\gamma_n-\gamma_n(1-\gamma_n)+2\beta_nL(b_nL+b_n+c_n) +\beta_nc_nL+\beta_n(r_{in}-1)\}\|\xi_{n-1}(t)-p(t)\|^2 -2\beta_n(1-k)\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2+2[\beta_nc_nL+\gamma_n]M(t) \tag{15}$$

$$\|x_n-p\|^2 \leq [1+\delta_{in}]\|\xi_{n-1}(t)-p(t)\|^2-2\beta_n(1-k)\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|^2 +2[\beta_nc_nL+\gamma_n]M(t) \tag{16}$$

where

$$\delta_{in}(t) = 2\{\beta_n^2+2\beta_n\gamma_n-\gamma_n(1-\gamma_n)+2\beta_nL(b_nL+b_n+c_n)+\beta_nc_nL +\beta_n(r_{in}-1)\}$$

$$\sigma_n(t) = 2[\beta_nc_nL+\gamma_n]M(t).$$

From conditions $(ii)-(iv)$, $\sum_{n=1}^{\infty}\delta_{in}<\infty$ and $\sum_{n=1}^{\infty}\sigma_n(t)<\infty$. Thus using Lemma 1.1, it follows that $\lim_{n\to\infty}\|\xi_{n-1}(t)-p(t)\|$ exists $\forall t\in\Omega$ and if $\{\xi_n(t)\}$ converges to a point p of T_i then $\lim\limits_{n\to\infty}\|\xi_n(t)-p(t)\|=0$. Let

$$\lim_{n\to\infty}\|\xi_n(t)-p(t)\|=d. \tag{17}$$

Since $\{\xi_n(t)\}$ is a bounded sequence, it follows from (7) and (8) that

$$\lim_{n\to\infty}\|\eta_n(t)-p(t)\|=d$$

which implies that

$$\limsup_{n\to\infty}\|T_{i(n)}^{k(n)}(t,\eta_n(t))-p(t)\|\le\limsup_{n\to\infty}r_n(t)\|\eta_n(t))-p(t)\|\le d. \tag{18}$$

By Lemma 1.2 ,(9) and (10)

$$\lim_{n\to\infty}\|T_{i(n)}^{k(n)}(t,\eta_n(t))-\xi_n(t)\|=0. \tag{19}$$

Furthermore, let $\lambda_{in}=\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|=0$

$$\begin{aligned}
\|\xi_n(t) &- T_i(t,\xi_n(t))\|=\|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\\
&+T_{i(n)}^{k(n)}(t,\xi_n(t))-T_i(t,\xi_n(t))\|\\
&\le \|\xi_n(t)-T_{i(n)}^{k(n)}(t,\xi_n(t))\|+L\|T_{i(n)-1}^{k(n)-1}(t,\xi_n(t))-\xi_n(t)\|\\
&= \lambda_{in}+L\|T_{i(n)-1}^{k(n)-1}(t,\xi_n(t))-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\\
&+T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))-\xi_n(t)\|\\
&\le \lambda_{in}+L^2\|\xi_n(t))-\xi_{n-1}(t)\|+L\|T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))-\xi_n(t)\|\\
&= \lambda_{in}+L^2\|\alpha_n(\xi_{n-1}(t)-\xi_{n-1}(t))+\beta_n(T_{i(n)}^{k(n)}(t,\eta_n(t))-\xi_{n-1}(t))\\
&+\gamma_n(f_n(t)-\xi_{n-1}(t))\|+L\|\alpha_n(\xi_{n-1}(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t)))\\
&+\beta_n(T_{i(n)}^{k(n)}(t,\eta_n(t))-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t)))\\
&+\gamma_n(f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t)))\|
\end{aligned}$$

$$
\begin{aligned}
&\leq \lambda_{in}+\beta_n L^2\|T_{i(n)}^{k(n)}(t,\eta_n(t))-\xi_{n-1}(t)\|+\gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\
&\quad +\alpha_n L\|\xi_{n-1}(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\|+\beta_n L^2\|T_i(t,\eta_n(t))-\xi_{n-1}(t)\| \\
&\quad +\gamma_n L\|f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \\
&= \lambda_{in}+\alpha_n L\lambda_{in-1}+\beta_n L^2\|T_{i(n)}^{k(n)}(t,\eta_n(t))-\xi_{n-1}(t)\| \\
&\quad +\gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\|+\beta_n L^2\|T_i(t,\eta_n(t))-\xi_{n-1}(t)\| \\
&\quad +\gamma_n L\|f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \\
&= \lambda_{in}+\alpha_n L\lambda_{in-1}+\beta_n L^2\|T_{i(n)}^{k(n)}(t,\eta_n(t))-T_{i(n)-1}^{k(n)-1}(t,\xi_n(t)) \\
&\quad +T_{i(n)-1}^{k(n)-1}(t,\xi_n(t))-\xi_{n-1}(t)\|+\gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\
&\quad +\beta_n L^2\|T_i(t,\eta_n(t))-T_i(t,\xi_n(t))+T_i(t,\xi_n(t))-\xi_{n-1}(t)\| \\
&\quad +\gamma_n L\|f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \\
&\leq \lambda_{in}+\alpha_n L\lambda_{in-1}+\beta_n L^3\|\eta_n(t)-\xi_n(t)\| \\
&\quad +\beta_n L^2\|T_{i(n)}^{k(n)}(t,\xi_n(t))-\xi_{n-1}(t)\|+\gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\
&\quad +\beta_n L^3\|\eta_n(t)-\xi_n(t)\|+\beta_n L^2\|T_i(t,\xi_{n-1}(t))-\xi_{n-1}(t)\| \\
&\quad +\gamma_n L\|f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \\
&= \lambda_{in}+\alpha_n L\lambda_{in-1}+2\beta_n L^3\|\eta_n(t)-\xi_n(t)\| \\
&\quad +\beta_n L^2\|T_{i(n)}^{k(n)}(t,\xi_n(t))-\xi_n+\xi_n-\xi_{n-1}(t)\| \\
&\quad +\gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\|+\beta_n L^2\|T_i(t,\xi_{n-1}(t))-\xi_n \\
&\quad +\xi_n-\xi_{n-1}(t)\|+\gamma_n L\|f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \\
&\leq \lambda_{in}+\alpha_n L\lambda_{in-1}+2\beta_n L^3\|\eta_n(t)-\xi_n(t)\| \\
&\quad +\beta_n L^2\|T_{i(n)}^{k(n)}(t,\xi_n(t))-\xi_n\|+\beta_n L^2\|\xi_n-\xi_{n-1}(t)\| \\
&\quad +\gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\|+\beta_n L^2\|T_i(t,\xi_{n-1}(t))-\xi_n\| \\
&\quad +\beta_n L^2\|\xi_n-\xi_{n-1}(t)\|+\gamma_n L\|f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \\
&= \lambda_{in}+\alpha_n L\lambda_{in-1}+2\beta_n L^3\|\eta_n(t)-\xi_n(t)\|+\beta_n L^2\lambda_{in} \\
&\quad +2\beta_n L^2\|\xi_n-\xi_{n-1}(t)\|+\gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\
&\quad +\beta_n L^2\|T_i(t,\xi_{n-1}(t))-\xi_n\|+\gamma_n L\|f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\|
\end{aligned}
$$

$$
\begin{aligned}
(1-\beta_n L^2)\|\xi_n(t) &- T_i(t,\xi_n(t))\| \le (1+\beta_n L^2)\lambda_{in} + \alpha_n L\lambda_{in-1} \\
&+2\beta_n L^3\|\eta_n(t)-\xi_n(t)\| \\
&+2\beta_n L^2\|\xi_n - \xi_{n-1}(t)\| + \gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\
&+\gamma_n L\|f_n(t) - T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \qquad (20)
\end{aligned}
$$

Observe that

$$
\begin{aligned}
\|\eta_n(t)-\xi_n(t)\| &= \|(a_n\xi_n + b_n T_{i(n)}^{k(n)}(t,\xi_n(t)) + c_n g_n(t)) - \xi_n(t)\| \\
&\le b_n\|T_{i(n)}^{k(n)}(t,\xi_n(t)) - \xi_n(t)\| + c_n\|g_n(t)-\xi_n(t)\| \\
&= b_n\lambda_{in} + c_n\|g_n(t)-\xi_n(t)\| \qquad (21)
\end{aligned}
$$

substitude (21) into (20)

$$
\begin{aligned}
(1-\beta_n L^2)\|\xi_n(t) &- T_i(t,\xi_n(t))\| \le (1+\beta_n L^2)\lambda_{in} + \alpha_n L\lambda_{in-1} \\
&+2\beta_n L^3\{b_n\lambda_{in} + c_n\|g_n(t)-\xi_n(t)\|\} \\
&+2\beta_n L^2\|\xi_n - \xi_{n-1}(t)\| + \gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\
&+\gamma_n L\|f_n(t) - T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
(1-\beta_n L^2)\|\xi_n(t) &- T_i(t,\xi_n(t))\| \le (1+\beta_n L^2 + 2\beta_n b_n L^3)\lambda_{in} \\
&+\alpha_n L\lambda_{in-1} + 2\beta_n c_n L^3\|g_n(t)-\xi_n(t)\| \\
&+2\beta_n L^2\|\xi_n - \xi_{n-1}(t)\| + \gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\
&+\gamma_n L\|f_n(t) - T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
\|\xi_n(t) &- T_i(t,\xi_n(t))\| \le \frac{1}{(1-\beta_n L^2)}\left[(1+\beta_n L^2 + 2\beta_n b_n L^3)\lambda_{in}\right. \\
&+\alpha_n L\lambda_{in-1} + 2\beta_n c_n L^3\|g_n(t)-\xi_n(t)\| \\
&+2\beta_n L^2\|\xi_n - \xi_{n-1}(t)\| + \gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\
&\left.+\ \gamma_n L\|f_n(t) - T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\|\right]
\end{aligned}
$$

$$\begin{aligned}\|\xi_n(t) \quad - \quad & T_i(t,\xi_n(t))\| \leq 2(1+\beta_n L^2+4\beta_n b_n L^3)\lambda_{in}+2\alpha_n L\lambda_{in-1} \\ & +4\beta_n c_n L^3\|g_n(t)-\xi_n(t)\|+4\beta_n L^2\|\xi_n-\xi_{n-1}(t)\| \\ & +2\gamma_n L^2\|f_n(t)-\xi_{n-1}(t))\| \\ & +2\gamma_n L\|f_n(t)-T_{i(n)-1}^{k(n)-1}(t,\xi_{n-1}(t))\| \end{aligned} \tag{24}$$

It follows that $\lim_{n\to\infty}\|\xi_n(t)-T_i(t,\xi_n(t))\|=0 \quad \forall i\in\mathscr{I}$.
Since one member of $\{T_{i(n)}^{k(n)}\}_{i=1}^{N}$ is semicompact, then, there exists a sbsequence $\{\xi_{n_j}(t)\}_{n=1}^{\infty}$ of $\{\xi_n(t)\}_{n=1}^{\infty}$ such that $\{\xi_{n_j}(t)\}_{n=1}^{\infty}$ converges strongly to u. Since K is closed, $u\in K$ and furthermore,

$$\|u-T_i(t,u)\|=\lim_{n\to\infty}\|\xi_{n_j}(t)-T_i(t,\xi_{n_j}(t))\|=0 \tag{25}$$

Thus $u\in F$. Since $\{\xi_{n_j}(t)\}_{n=1}^{\infty}$ converges strongly to u and $\lim_{n\to\infty}\|\xi_n(t)-u\|$ exists, it follows from Lemma 1.1 that $\{\xi_n(t)\}_{n=1}^{\infty}$ converges to u and hence the complete the proof.

Remarks:

1. Our result complements and generalizes the result of Banaejee and Choudhury [5].
2. If we set $a_n=1$, $\gamma_n=c_n=0$ the iteration scheme (2) takes the implicit form

$$\xi_n(t)=\alpha_n\xi_{n-1}(t)+\beta_n T_{i(n)}^{k(n)}(t,\xi_n(t)) \tag{26}$$

In case of $N=1$ and $t\notin\Omega$, (26) becomes the implicit iteration process (see for example Sun [34]) given by

$$\xi_n=\alpha_n\xi_{n-1}+\beta_n T^k\xi_n \tag{27}$$

Conflict of Interests

The authors declare that there is no conflict of interests.

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