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COMMON FIXED POINT THEOREM IN INTUITIONISTIC FUZZY 2-METRIC SPACES

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Abstract: The aim of this paper is to obtain a Common Fixed Point Theorem in Intuitionistic Fuzzy 2-Metric Space for weakly compatible map mappings using general contractive condition and to establish a situation in which a collection of map has fixed point. Our result extends and generalized the definition of weakly Compatible map in Intuitionistic Fuzzy 2-Metric Space.

Keywords: Intuitionistic Fuzzy 2-Metric Space, Weakly Compatible.

Mathematics subject classification: 54H25, 47H10

Introduction (1):

The concept of fuzzy sets introduced by Zadeh [1] in 1965 has opened up a new direction in the history of Mathematics. A large number of renowned mathematicians worked with fuzzy sets in different branches of Mathematics, one such being the fuzzy metric spaces. In 1986, [10] Atanassov generalized fuzzy sets by introducing intuitionistic fuzzy sets. Park [8] introduced the concept of intutionstic fuzzy metri c space with the help of a continuous t-norm and a continuous t-co norm as a generalization of fuzzy metric space due to George and Veeramani [2] and Kramosil and Michalek [7], which is a milestone in developing Fixed point theory in intuitionistic fuzzy metric space. Turkoglu [4] gave a

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generalization of Jungck's common fixed point theorem [6] to intuitionistic fuzzy metric spaces.

Recently Mursaleen[13] and Lohani[13] introduced the concept of intuitionistic fuzzy 2-metric space (2009) and intuitionistic fuzzy 2-norm space. In this paper we have proved the common fixed point theorem in Intuitionistic Fuzzy 2 Metric Space for weakly compatible maps satisfying the general contractive condition.

Preliminaries (2):

Definition 2.1: T-norm / Fuzzy intersection

A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if * is satisfying the following conditions:

(a) * is commutative and associative;

(b) * is continuous;

(c) a *1 = a for all a ϵ¸ [0, 1];

(d) a * b ≤ c *d whenever a ≤ c and b ≤ d, and a, b, c, d ϵ [0, 1].

Example 2.1

Standard Intersection: a*b = min{a,b}

Definition 2.2: T-conorm / Fuzzy Union

A binary operation ◇ :[0, 1] × [0, 1] → [0, 1] is a continuous t-conorm if ◇ is satisfying the following conditions:

(a) ◇ is commutative and associative;

(b) ◇ is continuous;

(c) a ◇0 = a for all a ϵ¸ [0, 1];

(d) a ◇ b ≤ c ◇d whenever a ≤ c and b ≤ d, and a, b, c, d ϵ [0, 1]

Example 2.2

Standard union: $a \Diamond b = \max\{a,b\}$

Definition 2.3:Intuitionistic Fuzzy 2 Metric Space

An 5-tuple $(X,M,N,*,\Diamond)$ is called intuitionistic fuzzy2-metric space if X is any non-empty set,* is a continuous t-norm, is a $\Diamond$ continuous t-co-norm and N,M are fuzzy sets on $X^3 \times (0,\infty)$; for all x, y, z,w ε X; s, t, r > 0,

(1)$M(x, y, z, t) + N(x, y, z, t) \leq 1$

(2)$M(x, y, z, t) > 0$

(3)$M(x, y, z, t) = 1$ if at least two of x, y, z are equal

(4)$M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t)$

(5)$M(x, y, w, t) * M(x, w, z, s) * M(w, y, z, r) \leq M(x, y, z, t + s + r)$

(6)$M(x, y, z, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous

(7)$N(x, y, z, t) < 1$

(8)$N(x, y, z, t) = 0$ if at least two of x, y, z are equal

(9)$N(x, y, z, t) = N(x, z, y, t) = N(y, z, x, t)$

(10)$N(x, y, w, t) \Diamond N(x, w, z, s) \Diamond N(w, y, z, r) \geq N(x, y, z, t + s + r)$

(11)$N(x, y, z, \cdot) : (0, \infty) \rightarrow (0, 1]$) is continuous.

Then (M;N) is said to be an intuitionistic fuzzy 2metric on X. The functions M(x,y,z,t) and N(x,y,z,t) denote the degree of nearness of membership and the degree of non- nearness of membership of (x,y,z,t) ε $X^3 \times (0,\infty)$respectively

Definition 2.4: Weakly Compatible Map

Two self maps A and S are said to be weakly compatible if they commute at their coincidence point i.e Ax=Sx implies that ASx=SAx.

Lemma 2.1 :

In an Intuitionistic fuzzy 2 metric space $(X,M,N,*,\Diamond)$ a sequence $\{x_n\}$ exist such that for all n Ɛ N , 0<k<1 and t>0

$M(x_n, x_n, x_{n+1}, k^n t) \geq M(x_0, x_0, x_1, t)$ and $N(x_n, x_n, x_{n+1}, k^n t) \leq N(x_0, x_0, x_1, t)$

then sequence $\{x_n\}$ is a cauchy sequence

proof Refer to [5].

Lemma 2.2 :

Let (X,M,N,*,◊) be Intuitionistic fuzzy 2 metric space then Э q ε (0,1) such that $M(x,y,z,qt+0) \geq M(x,y,z,t)$ and $N(x,y,z,qt+0) \leq N(x,y,z,t)$ for all x,y,z ε X with $z \neq x$ and $z \neq y$ and $t>0$ then $x=y$

Proof Refer to [5]

Theorem 3.1

Let A,S,T be self mapping of an intuitionistic fuzzy 2 metric space (X,M,N,*,◊) satisfying

1 A(x) C S(x) ∩ T(x)

2 {A,S}and {A,T}are weakly compatible

3 $M(Ax,Ay,Az,kt) \geq \min\{ M(Sx,Ax,Sx,t), M(Sx,Sx,Ay,t),$
$[M(Ax,Ay,Az,t/3)*M(Ax,Az,Tz,t/3)*M(Az,Ay,Tz.t/3)]\}$

$N(Ax,Ay,Az,kt) \leq \max\{ N(Sx,Ax,Sx,t), N(Sx,Sx,Ay,t),$
$[N(Ax,Ay,AZ,t/3)\Diamond N(Ax,Az,Tz,t/3)\Diamond N(Az,Ay,Tz.t/3)]\}$

For all x,y,z ε X then A,S,T have unique common fixed point .

Proof:

Let x_0 ε X be any arbitrary point.Since A(x) C S(x) therefore there exist a point x_1 in X such that $Ax_0 = S x_1$ also, A(x) C T(x) therefore there exist a point x_2 in X such that $Ax_1 = T x_2$ .In general we get the sequence $\{ y_n \}$ recursively as

$$Ax_{2n} = Sx_{2n+1} = y_{2n}$$

$$Ax_{2n+1} = Tx_{2n+2} = y_{2n+1}$$

Now we will prove that $\{ y_n \}$is a Cauchy sequence

Put $x=x_{2n}$,$y=x_{2n}$ & $z=x_{2n+1}$ in (3) we have

$$M(Ax_{2n},Ax_{2n},Ax_{2n+1},kt)\geq\min\{M(Sx_{2n},Ax_{2n},Sx_{2n},t),M(Sx_{2n},Sx_{2n},Ax_{2n},t),$$
$$[M(Ax_{2n},Ax_{2n},Ax_{2n+1},t/3)*M(Ax_{2n},Ax_{2n+1},Tx_{2n+1},t/3)*M(Ax_{2n+1},Ax_{2n},Tx_{2n+1}.t/3)]\}$$

$$N(Ax_{2n},Ax_{2n},Ax_{2n+1},kt)\leq\max\{N(Sx_{2n},Ax_{2n},Sx_{2n},t),N(Sx_{2n},Sx_{2n},Ax_{2n},t),$$
$$[N(Ax_{2n},Ax_{2n},Ax_{2n+1},t/3)\lozenge N(Ax_{2n},Ax_{2n+1},Tx_{2n+1},t/3)\lozenge N(Ax_{2n+1},Ax_{2n},Tx_{2n+1}.t/3)]\}$$

$$M(y_{2n},y_{2n},y_{2n+1},kt)\geq\min\{M(y_{2n-1},y_{2n-1},y_{2n},t),M(y_{2n-1},y_{2n-1},y_{2n},t),$$
$$[M(y_{2n},y_{2n},y_{2n+1},t/3)*M(y_{2n},y_{2n+1},y_{2n},t/3)*M(y_{2n+1},y_{2n},y_{2n}.t/3)]\}$$

$$N(y_{2n},y_{2n},y_{2n+1},kt)\leq\max\{N(y_{2n-1},y_{2n-1},y_{2n},t),N(y_{2n-1},y_{2n-1},y_{2n},t),$$
$$[N(y_{2n},y_{2n},y_{2n+1},t/3)\lozenge N(y_{2n},y_{2n+1},y_{2n},t/3)\lozenge N(y_{2n+1},y_{2n},y_{2n}.t/3)]\}$$

$$M(y_{2n},y_{2n},y_{2n+1},kt) \geq \min \{M(y_{2n-1},y_{2n-1},y_{2n},t),M(y_{2n-1},y_{2n-1},y_{2n},t),M(y_{2n},y_{2n},y_{2n},t)\}$$

$$N(y_{2n},y_{2n},y_{2n+1},kt) \leq \max \{N(y_{2n-1},y_{2n-1},y_{2n},t),N(y_{2n-1},y_{2n-1},y_{2n},t),N(y_{2n},y_{2n},y_{2n},t)\}$$

$$M(y_{2n},y_{2n},y_{2n+1},kt) \geq \min \{M(y_{2n-1},y_{2n-1},y_{2n},t),M(y_{2n-1},y_{2n-1},y_{2n},t),1\}$$

$$N(y_{2n},y_{2n},y_{2n+1},kt) \leq \max \{N(y_{2n-1},y_{2n-1},y_{2n},t),N(y_{2n-1},y_{2n-1},y_{2n},t),1\}$$

$$M(y_{2n},y_{2n},y_{2n+1},kt) \geq M(y_{2n-1},y_{2n-1},y_{2n},t)$$

$$N(y_{2n},y_{2n},y_{2n+1},kt) \leq N(y_{2n-1},y_{2n-1},y_{2n},t)$$

Now by lemma 2.1 we have :

$\{ y_n \}$is a Cauchy sequence

Hence its subsequence

$$\lim_{n\to\infty}Ax_{2n} = \lim_{n\to\infty}Sx_{2n+1} = \lim_{n\to\infty} Ax_{2n+1} = \lim_{n\to\infty}Tx_{2n+2} = z$$

Since A(x) C S(x) therefore there exist p ε X such that $p=S^{-1} z$

Put $x=p$,$y=x_{2n}$ & $z=x_{2n+1}$ in (3) we have

$$M(Ap,Ax_{2n},Ax_{2n+1},kt)\geq\min\{M(Sp,Ap,Sp,t),M(Sp,Sp,Ax_{2n},t),[M(Ap,Ax_{2n},Ax_{2n+1},t/3)* M(Ap,Ax_{2n+1},Tx_{2n+1},t/3)*M(Ax_{2n+1},Ax_{2n},Tx_{2n+1}.t/3)]\}$$

$$N(Ap,Ax_{2n},Ax_{2n+1},kt)\leq\max\{N(Sp,Ap,Sp,t),N(Sp,Sp,Ax_{2n},t),[N(Ap,Ax_{2n},Ax_{2n+1},t/3)\Diamond N(Ap,Ax_{2n+1},Tx_{2n+1},t/3)\Diamond N(Ax_{2n+1},Ax_{2n},Tx_{2n+1}.t/3)]\}$$

$$M(Ap,z,z,kt) \geq \min \{M(z,Ap,z,t),M(z,z,z,t),[M(Ap,z,z,t/3)*M(Ap,z,z,t/3)*M(z,z,z,t/3)]\}$$

.

$$N(Ap,z,z,kt) \leq \max \{N(z,Ap,z,t),N(z,z,z,t),[N(Ap,z,z,t/3)\Diamond N(Ap,z,z,t/3)\Diamond N(z,z,z,t/3)]\}$$

$$M(Ap,z,z,kt) \geq \min \{M(z,Ap,z,t),1,M(Ap,z,z,t)\}$$

$$N(Ap,z,z,kt) \leq \max \{N(z,Ap,z,t),1,N(Ap,z,z,t)\}$$

$M(Ap,z,z,kt) \geq M(Ap,z,z,t)$ and $N(Ap,z,z,kt) \leq N(Ap,z,z,t)$

Hence $Ap = z$

Therefore $Sp = Ap = z$

Since $\{A,S\}$ is weakly Compatible and let p Ɛ X be their coincidence point therefore,

$ASp=SAp$

Thus we have $Az = Sz$

Put $x=z$,$y=x_{2n}$ & $z=x_{2n+1}$ in (3) we have

$$M(Az,Ax_{2n},Ax_{2n+1},kt)\geq\min\{M(Sz,Az,Sz,t),M(Sz,Sz,Ax_{2n},t),[M(Az,Ax_{2n},Ax_{2n+1},t/3)* M(Az,Ax_{2n+1},Tx_{2n+1},t/3)*M(Ax_{2n+1},Ax_{2n},Tx_{2n+1}.t/3)]\}$$

$N(Az,Ax_{2n},Ax_{2n+1},kt) \leq \max\{N(Sz,Az,Sz,t),N(Sz,Sz,Ax_{2n},t),[N(Az,Ax_{2n},Ax_{2n+1},t/3)\Diamond$
$N(Az,Ax_{2n+1},Tx_{2n+1},t/3)\Diamond N(Ax_{2n+1},Ax_{2n},Tx_{2n+1}.t/3)]\}$

$M(Az,z,z,kt) \geq \min \{M(Az,Az,Az,t),M(Az,Az,z,t),M(Az,z,z,t)\}$

$N(Az,z,z,kt) \leq \max \{N(Az,Az,Az,t),N(Az,Az,z,t),N(Az,z,z,t)\}$

$M(Az,z,z,kt) \geq \min \{1, M(Az,Az,z,t),M(Az,z,z,t)\}$

$N(Az,z,z,kt) \leq \max \{1, N(Az,Az,z,t),N(Az,z,z,t)\}$

$M(Az,z,z,kt) \geq M(Az,z,z,t)$

$N(Az,z,z,kt) \leq N(Az,z,z,t)$

Hence we have $Az=z$

$Az = Sz = z$

Since A(x) C T(x) therefore there exist q ε X such that $q = T^{-1} z$

$Tq = z$

Put $x=x_{2n}$,$y=q$ & $z = x_{2n+1}$ in (3) we have

$M(Ax_{2n},Aq,Ax_{2n+1},kt) \geq \min\{M(Sx_{2n},Ax_{2n},Sx_{2n},t),M(Sx_{2n},Sx_{2n},Aq,t),$
$[M(Ax_{2n},Aq,Ax_{2n+1},t/3)*M(Ax_{2n},Ax_{2n+1},Tx_{2n+1},t/3)*M(Ax_{2n+1},Aq,Tx_{2n+1}.t/3)]\}$

$N(Ax_{2n},Aq,Ax_{2n+1},kt) \leq \max\{N(Sx_{2n},Ax_{2n},Sx_{2n},t),N(Sx_{2n},Sx_{2n},Aq,t),$
$[N(Ax_{2n},Aq,Ax_{2n+1},t/3)\Diamond N(Ax_{2n},Ax_{2n+1},Tx_{2n+1},t/3)\Diamond N(Ax_{2n+1},Aq,Tx_{2n+1}.t/3)]\}$

$M(z,Aq,z,kt) \geq \min\{ M(z,z,z,t),M(z,z,Aq,t),[M(z,Aq,z,t/3)*M(z,z,z,t/3)*M(z,Aq,z.t/3)]\}$

$N(z,Aq,z,kt) \leq \max \{N(z,z,z,t),N(z,z,Aq,t),[N(z,Aq,z,t/3)\Diamond N(z,z,z,t/3)\Diamond N(z,Aq,z.t/3)]\}$

$M(z,Aq,z,kt) \geq \min\{1,M(z,z,Aq,t),M(z,Aq,z,t)\}$

$N(z,Aq,z,kt) \leq \max\{1,N(z,z,Aq,t),N(z,Aq,z,t)\}$

$M(z,Aq,z,kt) \geq M(z,Aq,z,t)$

$N(z,Aq,z,kt) \leq N(z,Aq,z,t)$

Hence $Aq = z$

Since $\{A,T\}$ is weakly Compatible and let q Ɛ X be their coincidence point therefore,

$ATq=TAq$

Thus we have $Az = Tz$

Therefore $Az = Tz = Sz = z$

Thus z is the unique common fixed point of A , S , T .

Uniqueness

Let w be any other fixed point of A,S,T then

Put $x=z$,$y=w$ & $z = x_{2n+1}$ in (3) we have

$$M(Az,Aw,Ax_{2n+1},kt)\geq\min\{M(Sz,Az,Sz,t),M(Sz,Sz,Aw,t),[M(Az,Aw,Ax_{2n+1},t/3)* M(Az,Ax_{2n+1},Tx_{2n+1},t/3)*M(Ax_{2n+1},Az,Tx_{2n+1}.t/3)]\}$$

$$N(Az,Aw,Ax_{2n+1},kt)\leq\max\{N(Sz,Az,Sz,t),N(Sz,Sz,Aw,t),[N(Az,Aw,Ax_{2n+1},t/3)\lozenge N(Az,Ax_{2n+1},Tx_{2n+1},t/3)\lozenge N(Ax_{2n+1},Az,Tx_{2n+1}.t/3)]\}$$

$$M(z,w,z,kt) \geq \min\{ M(z,z,z,t),M(z,z,w,t),[M(z,w,z,t/3)*M(z,z,z,t/3)*M(z,z,z.t/3)]\}$$

$$N(z,w,z,kt) \leq \max \{N(z,z,z,t),N(z,z,w,t),[N(z,w,z,t/3)\lozenge N(z,z,z,t/3)\lozenge N(z,z,z.t/3)]\}$$

$$M(z,w,z,kt) \geq \min\{ 1,M(z,z,w,t),M(z,w,z,t)\}$$

$$N(z,w,z,kt) \leq \max\{ 1,N(z,z,w,t),N(z,w,z,t)\}$$

$$M(z,w,z,kt) \geq M(z,w,z,t)$$

$$N(z,w,z,kt) \leq N(z,w,z,t)$$

Thus z = w

Hence z is the unique common fixed point of A,S and T.

Conflict of Interests

The author declares that there is no conflict of interests.

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